

# CP violation in polarized $B \rightarrow \pi \ell^+ \ell^-$ and $B \rightarrow \rho \ell^+ \ell^-$ decays

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## Abstract

We study the decay rate and the CP violating asymmetry of the exclusive  $B \rightarrow \pi \ell^+ \ell^-$  and  $B \rightarrow \rho \ell^+ \ell^-$  decays in the case where one of the final leptons is polarized. We calculate the contributions coming from the individual polarization states in order to identify a so-called wrong sign decay, which is a decay with a given polarization, whose width and CP asymmetry are smaller as compared to the unpolarized one. The results are presented for electron and tau leptons. We observe that in particular decay channels, one can identify a wrong sign decay which is more sensitive to new physics beyond the Standard Model.

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## I. INTRODUCTION

The rare B decays, which are induced at quark level by flavour changing neutral currents (FCNC), have received a lot of attention, since they are very promising for investigating the Standard Model (SM) and searching for the new physics beyond it. Among these B-decays, the rare semileptonic ones have played a central role for a long time, since they offer the most direct methods to determine the weak mixing angles and Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. These decays can also be very useful to test the various new physics scenarios like the two Higgs doublet models (2HDM), minimal supersymmetric standard model (MSSM) [1]etc.

From experimental side, there is an impressive effort to search for B-decays, in B-factories such as Belle, BaBar, LHC-B. CLEO Collaboration reports for the branching ratios (BR) of the  $B^0 \rightarrow \pi^- \ell^+ \nu$  and  $B^0 \rightarrow \rho^- \ell^+ \nu$  decays [2] as

$$\begin{aligned} BR(B^0 \rightarrow \pi^- \ell^+ \nu) &= (1.8 \pm 0.4 \pm 0.3 \pm 0.2) \times 10^{-4}, \\ BR(B^0 \rightarrow \rho^- \ell^+ \nu) &= (2.57 \pm 0.29_{-0.46}^{+0.33} \pm 0.41) \times 10^{-4}. \end{aligned} \quad (1)$$

From these results, the value of the CKM matrix element  $|V_{ub}| = 3.25 \pm 0.14_{-0.29}^{+0.21} \pm 0.55$  has been determined [2]. Recently, the BR of the inclusive  $B \rightarrow X_s \ell^+ \ell^-$  decay has been also reported by Belle Collaboration [3];

$$BR(B \rightarrow X_s \ell^+ \ell^-) = (6.1 \pm 1.4_{-1.1}^{+1.4}) \times 10^{-6}, \quad (2)$$

which is very close to the value predicted by the SM [4]. The experimental result from BaBar collaboration for this BR is [5]:

$$BR(B \rightarrow X_s \ell^+ \ell^-) = (6.3 \pm 1.6_{-1.5}^{+1.8}) \times 10^{-6}. \quad (3)$$

From theoretical point of view exclusive channels are harder to evaluate than inclusive channels, because exclusive channels require an additional knowledge about form factors, which are used to incorporate hadronic effects. However, the exclusive channels are easier to measure. The decay channels that are induced by the  $b \rightarrow d \ell^+ \ell^-$  decay at the quark level are promising for searching the CP violation. For the B decays that are induced by the decay  $b \rightarrow s \ell^+ \ell^-$ , the terms which describe virtual effects as  $t\bar{t}$ ,  $c\bar{c}$  and  $u\bar{u}$  loops are in the matrix element proportional to  $V_{tb}V_{ts}^*$ ,  $V_{ub}V_{us}^*$  and  $V_{cb}V_{cs}^*$  respectively. Because of the unitary

property of the CKM matrix and because of the fact that  $V_{ub}V_{us}^*$  is small compared to the other CKM factors, the CP violation is strongly suppressed in these decays [6, 7]. Although the BR of the B decays induced by  $b \rightarrow d\ell^+\ell^-$  are smaller, the CKM factors  $V_{tb}V_{td}^*$ ,  $V_{ub}V_{ud}^*$  and  $V_{cb}V_{cd}^*$  are all of the same order. Therefore CP violation is much more considerable in these decays [8]. In this context, the exclusive  $B_d \rightarrow (\pi, \rho, \eta, \eta') \ell^+\ell^-$ , and  $B_d \rightarrow \gamma \ell^+\ell^-$  decays have been extensively studied in the SM [9]-[11] and beyond [12]-[16].

In [17], it has been observed that the unpolarized CP asymmetry and decay width for the inclusive  $b \rightarrow d\ell^+\ell^-$  decay are comparable to the CP asymmetry and decay width when one of the leptons is in a specific polarization state. The CP asymmetry as well as the decay rate in the case of the other polarization state turn out to be smaller as compared to the unpolarized spectrum and in [17] this is defined as the *wrong sign* polarized state. Along this line, in [18], a similar analysis about the CP asymmetries in  $b \rightarrow d\ell^+\ell^-$  decays has been performed in a model independent way and it was reported that polarized asymmetries are very sensitive to various new Wilson coefficients. In this paper, motivated by the works in [17, 18], we make a similar analysis to the exclusive  $B \rightarrow \pi\ell^+\ell^-$  and  $B \rightarrow \rho\ell^+\ell^-$  channels and calculate the contributions coming from the individual polarization states in order to identify a wrong sign decay. This feature can provide measurements involving a new physics search.

Our paper is organized as follows. In section II we present the effective Hamiltonian and derive the expressions for the unpolarized and the polarized differential decay rates of  $B \rightarrow \pi\ell^+\ell^-$  and  $B \rightarrow \rho\ell^+\ell^-$ . The CP violating asymmetries for these decays in the unpolarized as well as in the polarized case are calculated in section III. The numerical results and the discussions are presented in section IV which is followed by a conclusion section.

## II. EXCLUSIVE $B \rightarrow \pi \ell^+ \ell^-$ AND $B \rightarrow \rho \ell^+ \ell^-$ DECAYS

### A. Effective Hamiltonian

The leading order QCD corrected effective Hamiltonian, which is induced by the corresponding quark level process  $b \rightarrow d \ell^+ \ell^-$ , is given by [19]-[22]:

$$\mathcal{H}_{eff} = \frac{4G_F \alpha}{\sqrt{2}} V_{tb} V_{td}^* \left\{ \sum_{i=1}^{10} C_i(\mu) O_i(\mu) - \lambda_u \{ C_1(\mu) [O_1^u(\mu) - O_1(\mu)] \right. \\ \left. + C_2(\mu) [O_2^u(\mu) - O_2(\mu)] \} \right\}, \quad (4)$$

where

$$\lambda_u = \frac{V_{ub} V_{ud}^*}{V_{tb} V_{td}^*}, \quad (5)$$

using the unitarity of the CKM matrix i.e.,  $V_{tb} V_{td}^* + V_{ub} V_{ud}^* = -V_{cb} V_{cd}^*$ . The explicit forms of the operators  $O_i$  can be found in refs. [19, 22]. In Eq.(4),  $C_i(\mu)$  are the Wilson coefficients calculated at a renormalization point  $\mu$  and their evolution from the higher scale  $\mu = m_W$  down to the low-energy scale  $\mu = m_b$  is described by the renormalization group equation. For  $C_7^{eff}$  this calculation is performed upto next-to-next-to-leading logarithmic (NNLL) order in refs.[23]-[25], while  $C_9^{eff}$  and  $C_{10}$  were calculated in [26]. In the context of the SM NNLL QCD corrections to the  $BR$  [26]-[30] and the forward-backward asymmetry [31]- [34] in  $B \rightarrow X_s \ell^+ \ell^-$  are also available. For a recent review see e.g. [35]. The corresponding NNLL results for  $B \rightarrow X_d \ell^+ \ell^-$  are given in [36].

The term that is the source of the CP violation can be parameterized as follows:

$$C_9^{eff} = \xi_1 + \lambda_u \xi_2, \quad (6)$$

where

$$\xi_1 = C_9 + g(\hat{m}_c, s)(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) \\ - \frac{1}{2}g(\hat{m}_d, s)(C_3 + C_4) - \frac{1}{2}g(\hat{m}_b, s)(4C_3 + 4C_4 + 3C_5 + C_6) \\ + \frac{2}{9}(3C_3 + C_4 + 3C_5 + C_6), \quad (7)$$

and

$$\xi_2 = [g(\hat{m}_c, s) - g(\hat{m}_u, s)](3C_1 + C_2). \quad (8)$$

In Eqs.(7) and (8),  $s = q^2/m_B^2$  where  $q$  is the momentum transfer and  $\hat{m}_q = m_q/m_b$ . The functions  $g(\hat{m}_q, s)$  arise from one loop contributions of the four-quark operators  $O_1 - O_6$  and are given by

$$g(\hat{m}_q, s) = -\frac{8}{9} \ln \hat{m}_q + \frac{8}{27} + \frac{4}{9}y \quad (9)$$

$$-\frac{2}{9}(2+y)|1-y|^{1/2} \begin{cases} \left( \ln \left| \frac{\sqrt{1-y}+1}{\sqrt{1-y}-1} \right| - i\pi \right), & \text{for } y \equiv \frac{4\hat{m}_q^2}{s} < 1 \\ 2 \arctan \frac{1}{\sqrt{y-1}}, & \text{for } y \equiv \frac{4\hat{m}_q^2}{s} > 1. \end{cases}$$

$C_9^{eff}$  term receives also contributions from long-distance effects. The  $c\bar{c}$  resonance can be parameterized by means of a Breit-Wigner shape [37]. It is incorporated in the  $C_9^{eff}$  term by the following replacement,

$$g(\hat{m}_c, s) \rightarrow g(\hat{m}_c, s) - \frac{3\pi}{\alpha^2} \kappa \sum_{V=J/\psi, \psi', \dots} \frac{m_V Br(V \rightarrow \ell^+ \ell^-) \Gamma_{total}^V}{sm_B^2 - m_V^2 + im_V \Gamma_{total}^V}. \quad (10)$$

To reproduce the correct experimental BR for  $Br(B \rightarrow J/\psi X \rightarrow X \ell \bar{\ell}) = Br(B \rightarrow J/\psi X) Br(J/\psi \rightarrow X \ell \bar{\ell})$ , the factor  $\kappa$  is taken to be 2.3 [37].

Neglecting the mass of the  $d$  quark, the effective short distance Hamiltonian for the  $b \rightarrow d \ell^+ \ell^-$  decay in Eq.(4) leads to the QCD corrected matrix element:

$$\mathcal{M} = \frac{G_F \alpha}{2\sqrt{2}\pi} V_{tb} V_{td}^* \left\{ C_9^{eff}(m_b) \bar{d} \gamma_\mu (1 - \gamma_5) b \bar{\ell} \gamma^\mu \ell + C_{10}(m_b) \bar{d} \gamma_\mu (1 - \gamma_5) b \bar{\ell} \gamma^\mu \gamma_5 \ell \right. \\ \left. - 2C_7^{eff}(m_b) \frac{m_b}{q^2} \bar{d} i \sigma_{\mu\nu} q^\nu (1 + \gamma_5) b \bar{\ell} \gamma^\mu \ell \right\}. \quad (11)$$

## B. The exclusive $B \rightarrow \pi \ell^+ \ell^-$ decay

In this section we present the expressions for the differential decay rate of  $B \rightarrow \pi \ell^+ \ell^-$  decay with both unpolarized and polarized leptons. For this purpose, we need the following matrix elements, which are written in terms of the form factors:

$$\langle \pi(p_\pi) | \bar{d} \gamma_\mu (1 - \gamma_5) b | B(p_B) \rangle = f^+(q^2) (p_B + p_\pi)_\mu + f^-(q^2) q_\mu, \quad (12)$$

$$\langle \pi(p_\pi) | \bar{d} i \sigma_{\mu\nu} q^\nu (1 + \gamma_5) b | B(p_B) \rangle = [q^2 (p_B + p_\pi)_\mu - q_\mu (m_B^2 - m_\pi^2)] f_v(q^2). \quad (13)$$

Here,  $p_\pi$  and  $p_B$  are the four momenta of the  $\pi$  and the B meson, respectively. Also  $f^+$ ,  $f^-$  and  $f_v = -\frac{f_T}{m_B + m_\pi}$  represent the relevant form factors.

From Eq. (11), and using the matrix elements in Eqs. (12) and (13), we obtain the amplitude governing the  $B \rightarrow \pi \ell^+ \ell^-$  decay:

$$\mathcal{M}^{B \rightarrow \pi} = \frac{G_F \alpha}{2\sqrt{2}\pi} V_{tb} V_{td}^* \left\{ (2Ap_\pi^\mu + Bq^\mu) \bar{\ell} \gamma_\mu \ell + (2Gp_\pi^\mu + Dq^\mu) \bar{\ell} \gamma_\mu \gamma^5 \ell \right\}, \quad (14)$$

where

$$\begin{aligned} A &= C_9^{eff} f^+ - 2m_B C_7^{eff} f_v, \\ B &= C_9^{eff} (f^+ + f^-) + 2 \frac{m_B}{q^2} C_7^{eff} f_v (m_B^2 - m_\pi^2 - q^2), \\ G &= C_{10} f^+, \\ D &= C_{10} (f^+ + f^-). \end{aligned} \quad (15)$$

Using the matrix element in Eq. (14), performing summation over final lepton polarizations and integrating over angle variables, the unpolarized differential decay width is obtained as,

$$\left( \frac{d\Gamma^\pi}{ds} \right)_0 = \frac{G_F^2 \alpha^2}{2^{10} \pi^5} |V_{tb} V_{td}^*|^2 m_B^3 v \sqrt{\lambda_\pi} \Delta_\pi, \quad (16)$$

where

$$\begin{aligned} \Delta_\pi &= \frac{1}{3} m_B^2 \lambda_\pi (3 - v^2) (|A|^2 + |G|^2) + 16 m_\ell^2 r_\pi |G|^2 + 4 m_\ell^2 s |D|^2 \\ &\quad + 8 m_\ell^2 (1 - r_\pi - s) \text{Re}[GD^*], \end{aligned} \quad (17)$$

with  $r_\pi = m_\pi^2/m_B^2$ ,  $\lambda_\pi = r_\pi^2 + (s - 1)^2 - 2r_\pi(s + 1)$ ,  $v = \sqrt{1 - \frac{4t^2}{s}}$  and  $t = m_\ell/m_B$ .

In order to calculate the polarized decay spectrum, we need the final lepton polarizations. For this, one defines orthogonal unit vectors  $\vec{e}_L$ ,  $\vec{e}_T$  and  $\vec{e}_N$  such that in the rest frame of  $\ell^-$  lepton they are written as,

$$\begin{aligned} S_L^\mu &\equiv (0, \vec{e}_L) = \left( 0, \frac{\vec{p}_1}{|\vec{p}_1|} \right), \\ S_N^\mu &\equiv (0, \vec{e}_N) = \left( 0, \frac{\vec{k} \times \vec{p}_1}{|\vec{k} \times \vec{p}_1|} \right), \\ S_T^\mu &\equiv (0, \vec{e}_T) = \left( 0, \vec{e}_N^- \times \vec{e}_L^- \right). \end{aligned} \quad (18)$$

Here,  $\vec{p}_1$  is the 3-vector of  $\ell^-$  lepton and  $\vec{k}$  is the 3-vector of the final meson. The longitudinal unit vector  $S_L$  is boosted to the CM frame of  $\ell^+ \ell^-$  by Lorentz transformation:

$$S_{L,CM}^\mu = \left( \frac{|\vec{p}_1|}{m_\ell}, \frac{E_\ell \vec{p}_1}{m_\ell |\vec{p}_1|} \right), \quad (19)$$

while  $S_T$  and  $S_N$  are not changed by the boost. The differential decay rate of the  $B \rightarrow \pi \ell^+ \ell^-$  decay, for any spin direction  $\vec{n}$  of  $\ell^-$  can be written in the following form

$$\frac{d\Gamma^\pi(s, \vec{n})}{ds} = \frac{1}{2} \left( \frac{d\Gamma^\pi}{ds} \right)_0 [1 + P_i^\pi \vec{e}_i \cdot \vec{n}] , \quad (20)$$

where a sum over  $i = L, T, N$  is implied. Polarization components  $P_i^\pi$  in Eq. (20) are defined as

$$P_i^\pi(s) = \frac{d\Gamma^\pi(\vec{n} = \vec{e}_i)/ds - d\Gamma^\pi(\vec{n} = -\vec{e}_i)/ds}{d\Gamma^\pi(\vec{n} = \vec{e}_i)/ds + d\Gamma^\pi(\vec{n} = -\vec{e}_i)/ds} . \quad (21)$$

The resulting expressions for the polarization asymmetries are obtained as

$$\begin{aligned} P_L^\pi &= \frac{4m_B^2}{3\Delta_\pi} v \lambda_\pi \text{Re}[AG^*] , \\ P_T^\pi &= \frac{m_B^2}{\sqrt{s}\Delta_\pi} \sqrt{\lambda_\pi} \pi t \left( \text{Re}[AD^*]s + \text{Re}[AG^*](1 - r_\pi - s) \right) , \\ P_N^\pi &= 0 . \end{aligned} \quad (22)$$

Our results for  $P_L^\pi$  and  $P_T^\pi$  agree with the ones given in ref. [38]. As can be seen from the explicit expressions of  $P_i^\pi$ , the polarization  $P_T^\pi$  is proportional to  $m_\ell$  and therefore can be significant for  $\tau$  lepton only.

### C. The exclusive $B \rightarrow \rho \ell^+ \ell^-$ decay

In this section we present the expressions for the differential decay rate for  $B \rightarrow \rho \ell^+ \ell^-$  decay with both unpolarized and polarized leptons. For this, we need the following matrix elements:

$$\begin{aligned} \langle \rho(p_\rho, \varepsilon) | \bar{d} \gamma_\mu (1 - \gamma_5) b | B(p_B) \rangle &= -\epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} p_\rho^\lambda p_B^\sigma \frac{2V(q^2)}{m_B + m_\rho} - i\varepsilon_\mu^* (m_B + m_\rho) A_1(q^2) \\ &+ i(p_B + p_\rho)_\mu (\varepsilon^* q) \frac{A_2(q^2)}{m_B + m_\rho} + iq_\mu (\varepsilon^* q) \frac{2m_\rho}{q^2} [A_3(q^2) \\ &- A_0(q^2)] , \end{aligned} \quad (23)$$

$$\begin{aligned} \langle \rho(p_\rho, \varepsilon) | \bar{d} i \sigma_{\mu\nu} q^\nu (1 + \gamma_5) b | B(p_B) \rangle &= 4\epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} p_\rho^\lambda q^\sigma T_1(q^2) + 2i[\varepsilon_\mu^* (m_B^2 - m_\rho^2) \\ &- (p_B + p_\rho)_\mu (\varepsilon^* q)] T_2(q^2) + 2i(\varepsilon^* q) \\ &\left( q_\mu - (p_B + p_\rho)_\mu \frac{q^2}{m_B^2 - m_\rho^2} \right) T_3(q^2) , \end{aligned} \quad (24)$$

$$< \rho(p_\rho, \varepsilon) | \bar{d}(1 + \gamma_5)b | B(p_B) > = \frac{-1}{m_b} 2im_\rho(\varepsilon^* q) A_0(q^2), \quad (25)$$

where  $p_\rho$  and  $\varepsilon$  denote the four momentum and polarization vectors of the  $\rho$  meson, respectively.

From Eqs. (23-25), we get the following expression for the matrix element of the  $B \rightarrow \rho \ell^+ \ell^-$  decay:

$$\begin{aligned} \mathcal{M}^{B \rightarrow \rho} = & \frac{G_F \alpha}{2\sqrt{2}\pi} V_{tb} V_{td}^* \\ & \left\{ \bar{\ell} \gamma_\mu (1 - \gamma_5) \ell [2A \epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} p_\rho^\lambda p_B^\sigma + iB \varepsilon_\mu^* - iC(p_B + p_\rho)_\mu (\varepsilon^* q) - iD(\varepsilon^* q) q_\mu] \right. \\ & \left. + \bar{\ell} \gamma_\mu (1 + \gamma_5) \ell [2E \epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} p_\rho^\lambda p_B^\sigma + iF \varepsilon_\mu^* - iG(\varepsilon^* q)(p_B + p_\rho) - iH(\varepsilon^* q) q_\mu] \right\} \quad (26) \end{aligned}$$

where

$$\begin{aligned} A &= (C_9^{eff} - C_{10}) \frac{V}{m_B + m_\rho} + 4 \frac{m_b}{q^2} C_7^{eff} T_1, \\ B &= (m_B + m_\rho) \left( (C_9^{eff} - C_{10}) A_1 + \frac{4m_b}{q^2} (m_B^2 - m_\rho^2) C_7^{eff} T_2 \right), \\ C &= (C_9^{eff} - C_{10}) \frac{A_2}{m_B + m_\rho} + 4 \frac{m_b}{q^2} C_7^{eff} \left( T_2 + \frac{q^2}{m_B^2 - m_\rho^2} T_3 \right), \\ D &= 2(C_9^{eff} - C_{10}) \frac{m_\rho}{q^2} (A_3 - A_0) - 4C_7^{eff} \frac{m_b}{q^2} T_3, \\ E &= A(C_{10} \rightarrow -C_{10}), \\ F &= B(C_{10} \rightarrow -C_{10}), \\ G &= C(C_{10} \rightarrow -C_{10}), \\ H &= D(C_{10} \rightarrow -C_{10}). \end{aligned} \quad (27)$$

Here  $A_0, A_1, A_2, A_3, V, T_1, T_2$  and  $T_3$  are the relevant form factors.

Using the matrix element in Eq. (26), we find the unpolarized differential decay width as,

$$\left( \frac{d\Gamma^\rho}{ds} \right)_0 = \frac{\alpha^2 G_F^2 m_B}{2^{12} \pi^5} |V_{tb} V_{td}^*|^2 v \sqrt{\lambda_\rho} \Delta_\rho, \quad (28)$$



where

$$\begin{aligned}
\Delta_\rho = & \frac{8}{3}m_B^4\lambda_\rho \left[ (m_B^2s - m_\ell^2) (|A|^2 + |E|^2) + 6m_\ell^2 \operatorname{Re}(AE^*) \right] \\
& + 24m_\ell^2 \operatorname{Re}(BF^*) + \frac{1}{r_\rho}m_B^4m_\ell^2s\lambda_\rho |D - H|^2 \\
& + \frac{2}{r_\rho}m_B^2m_\ell^2\lambda_\rho \left( \operatorname{Re}[B(-D^* + G^* + H^*)] + \operatorname{Re}[F(C^* + D^* - H^*)] \right) \\
& + \frac{1}{2}m_\ell \operatorname{Re}[(C - G)(D^* - H^*)] - \frac{2}{r_\rho}m_B^4m_\ell^2\lambda_\rho(2 + 2r_\rho - s) \operatorname{Re}(CG^*) \\
& - \frac{2}{3r_\rho s}m_B^2\lambda_\rho \left[ m_\ell^2(2 - 2r_\rho + s) + m_B^2s(1 - r_\rho - s) \right] \left[ \operatorname{Re}(BC^*) + \operatorname{Re}(FG^*) \right] \\
& + \frac{1}{3r_\rho s} \left[ 2m_\ell^2(\lambda_\rho - 6r_\rho s) + m_B^2s(\lambda_\rho + 12r_\rho s) \right] (|B|^2 + |F|^2) \\
& + \frac{1}{3r_\rho s}m_B^4\lambda_\rho \left( m_B^2s\lambda_\rho + m_\ell^2[2\lambda_\rho + 3s(2 + 2r_\rho - s)] \right) (|C|^2 + |G|^2) , \tag{29}
\end{aligned}$$

where  $\lambda_\rho = r_\rho^2 + (s - 1)^2 - 2r_\rho(s + 1)$  and  $r_\rho = m_\rho^2/m_B^2$ .

The polarization components are obtained in the same way as in the previous section. The differential decay rate of the  $B \rightarrow \rho \ell^+ \ell^-$ -decay, for any spin direction  $\vec{n}$  of  $\ell^-$  can be written in the following form:

$$\frac{d\Gamma^\rho(s, \vec{n})}{ds} = \frac{1}{2} \left( \frac{d\Gamma^\rho}{ds} \right)_0 [1 + P_i^\rho \vec{e}_i \cdot \vec{n}] , \tag{30}$$

where a sum over  $i = L, T, N$  is implied. The resulting expressions for the polarization asymmetries are obtained as,

$$\begin{aligned}
P_L^\rho = & \frac{-1}{3r_\rho\Delta_\rho}m_B^2v \left( 8m_B^4sr_\rho\lambda_\rho(|E|^2 - |A|^2) - (12r_\rho s + \lambda_\rho)(|B|^2 - |F|^2) + m_B^4\lambda_\rho^2(|G|^2 - |C|^2) \right. \\
& \left. - 2m_B^2\lambda_\rho(-1 + r_\rho + s)\operatorname{Re}[CB^* - FG^*] \right) , \\
P_T^\rho = & \frac{-1}{4r_\rho\sqrt{s}\Delta_\rho}m_Bm_\ell\pi\sqrt{\lambda_\rho} \left( m_B^4\lambda_\rho(r_\rho - 1)(|C|^2 - |G|^2) + m_B^2s(1 + 3r_\rho - s)\operatorname{Re}[CF^* - BG^*] \right. \\
& + 8r_\rho sm_B^2\operatorname{Re}[(A + E)(B^* + F^*)] + m_B^2(\lambda_\rho + (-1 + r_\rho + s)(r_\rho - 1))\operatorname{Re}[BC^* - FG^*] \\
& + (-1 + r_\rho + s)(|B|^2 - |F|^2 + sm_B^2\operatorname{Re}[(B + F)(H^* - D^*)]) \\
& \left. + m_B^4s\lambda_\rho\operatorname{Re}[(C + G)(H^* - D^*)] \right) , \tag{31} \\
P_N^\rho = & \frac{1}{4r_\rho\Delta_\rho}m_B^3m_\ell\pi v\sqrt{s\lambda_\rho} \left( 8\operatorname{Im}[EB^* + FA^*] - (1 - r_\rho - s)\operatorname{Im}[(B - F)(D^* - H^*)] \right. \\
& \left. - (1 + 3r_\rho - s)\operatorname{Im}[(B - F)(C^* - G^*)] + m_B^2\lambda_\rho\operatorname{Im}[(C - G)(D^* - H^*)] \right) .
\end{aligned}$$

Our results for  $P_L^\rho$ ,  $P_N^\rho$  and  $P_T^\rho$  agree with those given in [39] for the SM case. As can be seen from the explicit expressions of  $P_i^\rho$ , they involve various quadratic combinations of the

Wilson coefficients and hence they are quite sensitive to the new physics. The polarizations  $P_N^\rho$  and  $P_T^\rho$  are again proportional to  $m_\ell$  as in the  $B \rightarrow \pi \ell^+ \ell^-$ -decay and therefore can be significant for  $\tau$  lepton only.

### III. CP VIOLATION

#### A. CP Violating Asymmetry in $B \rightarrow \pi \ell^+ \ell^-$ -decay

In  $B \rightarrow \pi \ell^+ \ell^-$ -decay with unpolarized final leptons, CP violating differential decay width asymmetry is defined as,

$$A_{CP}^\pi(s) = \frac{(d\Gamma^\pi/ds)_0 - (d\bar{\Gamma}^\pi/ds)_0}{(d\Gamma^\pi/ds)_0 + (d\bar{\Gamma}^\pi/ds)_0} = \frac{\Delta_\pi - \bar{\Delta}_\pi}{\Delta_\pi + \bar{\Delta}_\pi}, \quad (32)$$

where

$$\frac{d\Gamma^\pi}{ds} = \frac{d\Gamma(B \rightarrow \pi e^+ e^-)}{ds}, \quad \frac{d\bar{\Gamma}^\pi}{ds} = \frac{d\Gamma(\bar{B} \rightarrow \bar{\pi} e^+ e^-)}{ds}.$$

In the SM, the Wilson coefficient  $C_9^{eff}$  is the only one that contributes to  $A_{CP}$  above since it has an imaginary component as well as a real one, which can be parameterized as in Eq.(6). Therefore,  $(d\bar{\Gamma}^\pi/ds)_0$  and  $\bar{\Delta}_\pi$  in Eq. (32) can be obtained from  $(d\Gamma^\pi/ds)_0$  and  $\Delta_\pi$  by making the replacement,

$$\left(\frac{d\bar{\Gamma}^\pi}{ds}\right)_0 = \left(\frac{d\Gamma^\pi}{ds}\right)_0 \Big|_{\lambda_u \rightarrow \lambda_u^*}, \quad \bar{\Delta}_\pi = \Delta_\pi \Big|_{\lambda_u \rightarrow \lambda_u^*}. \quad (33)$$

Using Eqs. (17), (32) and (33), the CP violating asymmetry is obtained as

$$A_{CP}(s) = \frac{-2\text{Im}[\lambda_u]\Sigma_\pi(s)}{\Delta_\pi + 2\text{Im}[\lambda_u]\Sigma_\pi(s)},$$

where

$$\Sigma_\pi(s) = \frac{1}{3}m_B^2\lambda_\pi(3-v^2)(f^{+2}\text{Im}[\xi_1^*\xi_2] - 2m_B f_v f^+ \text{Im}[\xi_2 C_7^{\text{eff}*}]). \quad (34)$$

When one of the leptons is polarized in  $B \rightarrow \pi \ell^+ \ell^-$ -decay, CP violating asymmetry can be defined as follows:

$$A_{CP}^\pi(s, \vec{n}) = \frac{d\Gamma^\pi(s, \vec{n})/ds - d\bar{\Gamma}^\pi(s, \vec{n} = -\vec{n})/ds}{(d\Gamma^\pi/ds)_0 + (d\bar{\Gamma}^\pi/ds)_0}, \quad (35)$$

where

$$\frac{d\Gamma^\pi(s, \vec{n})}{ds} = \frac{d\Gamma^\pi(B \rightarrow \pi e^+ e^-(\vec{n}))}{ds}, \quad \frac{d\bar{\Gamma}^\pi(s, \vec{n})}{ds} = \frac{d\Gamma(\bar{B} \rightarrow \bar{\pi} e^+ e^-(\vec{n}))}{ds}.$$

Here,  $\vec{n}$  is the spin direction of the  $\ell^+$  in the  $\bar{B} \rightarrow \bar{\pi}\ell^+\ell^-$  decay. From the expression for the polarized differential decay width for the  $B \rightarrow \pi\ell^+\ell^-$  decay given by Eq. (20), the width for the corresponding CP conjugated process reads,

$$\frac{d\bar{\Gamma}^\pi(s, \vec{n})}{ds} = \frac{1}{2} \left( \frac{d\bar{\Gamma}^\pi}{ds} \right)_0 [1 + \bar{P}_i^\pi \vec{e}_i \cdot \vec{n}] . \quad (36)$$

Since in the CP conserving case  $\bar{P}_i^\pi = -P_i^\pi$ , in the general case with the choice  $\vec{e}_i = \vec{e}_i$ ,  $\bar{P}_i^\pi$  can be constructed with the replacement,

$$\bar{P}_i^\pi = -P_i^\pi |_{\lambda_u \rightarrow \lambda_u^*} . \quad (37)$$

Inserting Eqs. (20) and (36) into Eq. (35), and setting  $\vec{n} = \vec{n}$ , the CP violating asymmetry when lepton is polarized with  $\vec{n} = \pm \vec{e}_i$  is given by,

$$A_{CP}^\pi(s, \vec{n} = \pm \vec{e}_i) = \frac{1}{2} \frac{(d\Gamma^\pi/ds)_0 [1 \pm P_i^\pi] - (\bar{\Gamma}^\pi/ds)_0 [1 \pm \bar{P}_i^\pi]}{(d\Gamma^\pi/ds)_0 + (d\bar{\Gamma}^\pi/ds)_0} ,$$

or, by making use of the replacements in Eq. (33) and Eq. (37) we further obtain,

$$\begin{aligned} A_{CP}^\pi(s, \vec{n} = \pm \vec{e}_i) &= \frac{1}{2} \left\{ \frac{(d\Gamma^\pi/ds)_0 - (d\bar{\Gamma}^\pi/ds)_0}{(d\Gamma^\pi/ds)_0 - (d\bar{\Gamma}^\pi/ds)_0} \pm \frac{(d\Gamma^\pi/ds)_0 P_i^\pi - ((d\Gamma^\pi/ds)_0 P_i^\pi) |_{\lambda_u \rightarrow \lambda_u^*}}{(d\Gamma^\pi/ds)_0 - (d\bar{\Gamma}^\pi/ds)_0} \right\} \\ &= \frac{1}{2} \{ A_{CP}^\pi(s) \pm \delta A_{CP}^{\pi i}(s) \} . \end{aligned} \quad (38)$$

The  $\delta A_{CP}^{\pi i}(s)$  terms in Eq. (38) describe the modifications to the unpolarized decay width, which can be written as,

$$\delta A_{CP}^{\pi i}(s) = \frac{-2\text{Im}[\lambda_u] \delta \Sigma_\pi^i(s)}{\Delta_\pi(s) + 2\text{Im}[\lambda_u] \Sigma_\pi(s)} , \quad (39)$$

where

$$\delta \Sigma_\pi^L(s) = \frac{2}{3} m_B^2 v \lambda_\pi f^{+2} \text{Im}[\xi_2 C_{10}^*] , \quad (40)$$

$$\delta \Sigma_\pi^T(s) = \frac{m_B^2 t \pi \sqrt{\lambda_\pi}}{2\sqrt{s}} ((1 - r_\pi) f^{+2} + s f^+ f^-) \text{Im}[\xi_2 C_{10}^*] , \quad (41)$$

$$\delta \Sigma_\pi^N(s) = 0 . \quad (42)$$

## B. CP Violating Asymmetry in $B \rightarrow \rho\ell^+\ell^-$ decay

In  $B \rightarrow \rho\ell^+\ell^-$  decay with unpolarized final leptons, CP violating differential decay width asymmetry is defined as in Eq.(32) with the replacement  $\Delta_\pi \rightarrow \Delta_\rho$  and  $d\Gamma^\pi/ds \rightarrow d\Gamma^\rho/ds$ . Using Eqs. (29), (32) and (33), the CP violating asymmetry is given as,

$$A_{CP}^\rho(s) = \frac{-\text{Im}[\lambda_u] \Sigma_\rho(s)}{2\Delta_\rho + \text{Im}[\lambda_u] \Sigma_\rho(s)} ,$$

where

$$\begin{aligned}
\Sigma_\rho(s) = & \frac{4(s+2t)}{3r_\rho s(1+r_\rho)} \left\{ m_{B\rho} \text{Im}[\xi_2] \left[ A_1 m_{B\rho}^2 \left( c_2 m_B^2 (-1+r_\rho+s) \lambda_\rho + c_1 (12r_\rho s + \lambda_\rho) \right) \right. \right. \\
& + m_B^2 \lambda_\rho \left( 8m_\rho^2 r_\rho s V c_3 + A_2 (c_1 (-1+r_\rho+s) + c_2 m_B^2 \lambda_\rho) \right) \left. \right] \\
& - 2\text{Im}[\xi_1 \xi_2^*] \left[ 2A_1 A_2 m_B^2 m_{B\rho}^2 \lambda_\rho (-1+r_\rho+s) + A_1^2 m_{B\rho}^4 (12r_\rho s + \lambda_\rho) \right. \\
& + m_B^4 \lambda_\rho (8r_\rho s V^2 + A_2^2 \lambda_\rho) \left. \right] \left. \right\}, \tag{43}
\end{aligned}$$

with  $m_{B\rho} \equiv m_B + m_\rho$  and

$$\begin{aligned}
c_1 &= \frac{8m_b C_7^{eff}}{q^2} (m_B^2 - m_\rho^2) T_2, \quad c_2 = 8 \frac{m_b}{q^2} C_7^{eff} \left( T_2 + \frac{q^2}{m_B^2 - m_\rho^2} T_3 \right), \\
c_3 &= \frac{8m_b C_7^{eff}}{q^2} T_1. \tag{44}
\end{aligned}$$

When one of the leptons is polarized in  $B \rightarrow \rho \ell^+ \ell^-$  decay, CP violating asymmetry can be defined as follows:

$$A_{CP}^\rho(s, \vec{n}) = \frac{d\Gamma^\rho(s, \vec{n})/ds - d\bar{\Gamma}^\rho(s, \vec{n} = -\vec{n})/ds}{(d\Gamma^\rho/ds)_0 + (d\bar{\Gamma}^\rho/ds)_0}, \tag{45}$$

where

$$\frac{d\Gamma^\rho(s, \vec{n})}{ds} = \frac{d\Gamma(B \rightarrow \rho \ell^+ \ell^- (\vec{n}))}{ds}, \quad \frac{d\bar{\Gamma}^\rho(s, \vec{n})}{ds} = \frac{d\Gamma(\bar{B} \rightarrow \rho \ell^+ \ell^- (\vec{n}))}{ds}.$$

Here,  $\vec{n}$  is the spin direction of the  $\ell^+$  in the  $\bar{B} \rightarrow \rho \ell^+ \ell^-$  decay. From the expression for the polarized differential decay width in the  $B \rightarrow \rho \ell^+ \ell^-$  decay given by Eq. (30), the width for corresponding CP conjugated process reads

$$\frac{d\bar{\Gamma}^\rho(s, \vec{n})}{ds} = \frac{1}{2} \left( \frac{d\bar{\Gamma}^\rho}{ds} \right)_0 [1 + \bar{P}_i^\rho \vec{e}_i \cdot \vec{n}]. \tag{46}$$

Inserting Eqs. (30) and (46) into Eq. (45), and setting  $\vec{n} = \vec{n}$ , the CP violating asymmetry when lepton is polarized with  $\vec{n} = \pm \vec{e}_i$  is given by

$$A_{CP}^\rho(s, \vec{n} = \pm \vec{e}_i) = \frac{1}{2} \frac{(d\Gamma^\rho/ds)_0 [1 \pm P_i^\rho] - (\bar{\Gamma}^\rho/ds)_0 [1 \pm \bar{P}_i^\rho]}{(d\Gamma^\rho/ds)_0 + (d\bar{\Gamma}^\rho/ds)_0},$$

or, by making use of the replacements in Eq. (33) and Eq. (37) with  $\pi \rightarrow \rho$  we further obtain

$$\begin{aligned} A_{CP}^\rho(s, \vec{n} = \pm \vec{e}_i) &= \frac{1}{2} \left\{ \frac{(d\Gamma^\rho/ds)_0 - (d\bar{\Gamma}^\rho/ds)_0}{(d\Gamma^\rho/ds)_0 - (d\bar{\Gamma}^\rho/ds)_0} \pm \frac{(d\Gamma^\rho/ds)_0 P_i^\rho - ((d\Gamma^\rho/ds)_0 P_i^\rho) |_{\lambda_u \rightarrow \lambda_u^*}}{(d\Gamma^\rho/ds)_0 - (d\bar{\Gamma}^\rho/ds)_0} \right\} \\ &= \frac{1}{2} \{ A_{CP}^\rho(s) \pm \delta A_{CP}^{\rho i}(s) \} . \end{aligned} \quad (47)$$

The  $\delta A_{CP}^{\rho i}(s)$  terms in Eq. (47) describe the modifications to the unpolarized decay width, which can be written as

$$\delta A_{CP}^{\rho i}(s) = \frac{\text{Im}\lambda_u \delta \Sigma_\rho^i(s)}{\Delta_\rho(s) + \bar{\Delta}_\rho(s)} , \quad (48)$$

where

$$\begin{aligned} \delta \Sigma_\rho^L(s) &= \frac{4m_B v}{3r_\rho(1 + \sqrt{r_\rho})} \text{Im}[\xi_2] \left\{ A_1 m_{B\rho}^2 \left( c_2' m_B^2 (-1 + r + s) \lambda_\rho + c_1' (12r_\rho s + \lambda_\rho) \right) \right. \\ &\quad \left. + m_B^2 \lambda_\rho \left( 8m_\rho^2 r_\rho s V c_4' + A_2 (c_1' (-1 + r_\rho + s) + c_2' m_B^2 \lambda_\rho) \right) \right\} , \end{aligned} \quad (49)$$

$$\begin{aligned} \delta \Sigma_\rho^T(s) &= \frac{m_B^2 m_\ell \pi}{r_\rho(1 + \sqrt{r_\rho})} \frac{\sqrt{\lambda_\rho}}{\sqrt{s}} \left\{ -A_1 m_{B\rho}^2 \text{Im}[\xi_2] \left[ (-1 + r_\rho + s) c_1' / m_B^2 \right. \right. \\ &\quad \left. \left. + (-1 + r_\rho + s)(-1 + r_\rho) c_2' + s(8r_\rho c_3 + (-1 + r_\rho + s) c_3') \right] \right. \\ &\quad \left. + \text{Im}[\xi_2] \left[ 8r_\rho s V c_1 - A_2 \lambda_\rho (c_1' + m_B^2 ((r_\rho - 1) c_2' - s c_3')) \right] \right. \\ &\quad \left. - 32m_{B\rho} r_\rho s A_1 V \text{Im}[\xi_1 \xi_2^*] \right\} , \end{aligned} \quad (50)$$

$$\begin{aligned} \delta \Sigma_\rho^N(s) &= \frac{m_B^2 m_\ell \pi v}{2r_\rho(1 + \sqrt{r_\rho})} \sqrt{\lambda_\rho} \sqrt{s} \text{Re}[\xi_2] \left\{ (-A_2 D_1 + A_1 D_2 m_{B\rho}^2) (-1 - 3r_\rho + s) \right. \\ &\quad \left. - m_{B\rho} (-1 + r_\rho + s) (A_1 D_3 m_{B\rho} - 2D_1 T_3 / m_b) - 8r_\rho (A_1 c_4' m_{B\rho}^2 + c_1' V) \right\} , \end{aligned} \quad (51)$$

where

$$\begin{aligned} c_1' &= -2m_{B\rho} A_1 C_{10} \quad , \quad c_2' = -2A_2 C_{10} / m_{B\rho} , \\ c_3' &= -4T_3 C_{10} / m_b \quad , \quad c_4' = -2V C_{10} / m_{B\rho} , \end{aligned} \quad (52)$$

and

$$D_1 = F(C_9^{eff} \rightarrow 0) , \ D_2 = G(C_9^{eff} \rightarrow 0) , \ D_3 = H(C_9^{eff} \rightarrow 0) . \quad (53)$$

#### IV. NUMERICAL RESULTS AND DISCUSSION

In this section we present the numerical analysis of both the exclusive decays  $B \rightarrow \pi \ell^+ \ell^-$  and  $B \rightarrow \rho \ell^+ \ell^-$  for  $\ell = e, \tau$ . We do not present the results for  $\ell = \mu$  because they are similar to the ones for  $\ell = e$ . The input parameters we used in our numerical analysis are as follows:

$$\begin{aligned} m_B &= 5.28 \text{ GeV} , \ m_b = 4.8 \text{ GeV} , \ m_c = 1.4 \text{ GeV} , \ m_\tau = 1.78 \text{ GeV} , \ m_e = 0.511 \text{ MeV} , \\ m_\mu &= 0.106 \text{ GeV} , \ m_\pi = 0.14 \text{ GeV} , \ m_\rho = 0.77 \text{ GeV} , \ m_d = m_u = m_\pi = 0.14 \text{ GeV} , \\ |V_{cb}| &= 0.044 , \ \alpha^{-1} = 129 , \ G_f = 1.17 \times 10^{-5} \text{ GeV}^{-2} , \ \tau_B = 1.56 \times 10^{-12} \text{ s} . \end{aligned} \quad (54)$$

Using the Wolfenstein parametrization of the CKM matrix [40],  $\lambda_u$  in Eq. (6) can be written as:

$$\lambda_u = \frac{\rho(1-\rho) - \eta^2 - i\eta}{(1-\rho)^2 + \eta^2} + O(\lambda^2). \quad (55)$$

Furthermore, we use the relation

$$\frac{|V_{tb}V_{td}^*|^2}{|V_{cb}|^2} = \lambda^2[(1-\rho)^2 + \eta^2] + \mathcal{O}(\lambda^4) \quad (56)$$

where  $\lambda = \sin \theta_C \simeq 0.221$  and adopt the values of the Wolfenstein parameters as  $\rho = 0.25$  and  $\eta = 0.34$ .

In order to obtain numerical results for the  $B \rightarrow \pi \ell^+ \ell^-$  and  $B \rightarrow \rho \ell^+ \ell^-$  decays, we also need the numerical values of the decay form factors. The literature on this subject is very rich; we give some of them here. For  $B \rightarrow \pi(\rho)$  form factors are calculated in the constituent quark model [41] and using the light-cone QCD sum rules [42, 43] ([44, 45]). In [46] the results of the lattice QCD calculations are given for the  $B \rightarrow \pi, \rho$  form factors, while perturbative QCD approach [47] and the so-called large energy effective theory [48] have also been employed to calculate these form factors.

### A. Numerical results of the exclusive $B \rightarrow \pi \ell^+ \ell^-$ decay

In order to obtain numerical results for the  $B \rightarrow \pi \ell^+ \ell^-$  decay, we have made use of the results of the constituent quark model [41], where the form factors  $f_T$  and  $f_+$  can be parameterized as:

$$f(q^2) = \frac{f(0)}{(1 - q^2/M^2)[1 - \sigma_1 q^2/M^2 + \sigma_2 q^4/M^4]}. \quad (57)$$

In this model,  $f_-$  is redefined as:

$$F_0 = f_+ + \frac{q^2}{(p_B + p_\pi)q} f_-, \quad (58)$$

and its interpolation formula is given as:

$$f(q^2) = \frac{f(0)}{[1 - \sigma_1 q^2/M^2 + \sigma_2 q^4/M^4]}. \quad (59)$$

The parameters  $f(0)$ ,  $\sigma_1$  and  $\sigma_2$  can be found in Table I. Note that for  $f_+$  and  $f_T$  a simple monopole two-parameter formula is used *viz.*  $\sigma_2 = 0$ .

|       | $f(0)$ | $\sigma_1$ | $\sigma_2$ |
|-------|--------|------------|------------|
| $f_+$ | 0.29   | 0.48       |            |
| $F_0$ | 0.29   | 0.76       | 0.28       |
| $f_T$ | 0.28   | 0.48       |            |

TABLE I:  $B \rightarrow \pi$  transition form factors in the constituent quark model.

In Fig.(1) we present our results of the differential branching ratios (dBR/ds) of the unpolarized and longitudinally polarized  $B \rightarrow \pi e^+ e^-$  decay. dBR/ds for  $\vec{n} = -\vec{e}_L$  polarized case is close to the one of unpolarized decay, which implies that the decay is naturally left-handed. dBR/ds for the  $\vec{n} = +\vec{e}_L$  polarization case is far below dBR/ds for the unpolarized one. Thus,  $\vec{n} = +\vec{e}_L$  polarized  $B \rightarrow \pi e^+ e^-$  decay corresponds to a wrong sign decay.

In Figs.(2) and (3), we plot the longitudinally polarized asymmetries and the unpolarized CP violating asymmetry together with  $-\delta A_{CP}^L$  of the  $B \rightarrow \pi e^+ e^-$  decay, respectively. From Fig.(2) it can be observed that  $A_{CP}(\vec{n} = -\vec{e}_L)$  is much larger than  $A_{CP}(\vec{n} = +\vec{e}_L)$ . It is also observed from Fig.(3) that  $-\delta A_{CP}^L$  exceeds the unpolarized  $A_{CP}$  in some kinematical regions but is mostly comparable with it. Particularly, in the region  $(2m_\ell/m_B)^2 \leq s \leq$

$((m_{J/\psi} - 0.02)/m_B)^2$ , which is free of resonance contribution, we find that  $\delta A_{CP}^L$  and  $A_{CP}$  are about 6%. We see also from Fig.(3) that in the resonance region  $\delta A_{CP}^L$  can reach values up to 25%.

In Fig.(4), we present dBR/ds for the decay  $B \rightarrow \pi\tau^+\tau^-$  for unpolarized, longitudinally and transversely polarized  $\tau$  leptons. We observe that dBR/ds for  $\vec{n} = -\vec{e}_L$  and  $\vec{n} = -\vec{e}_T$  are close to unpolarized dBR/ds, while it becomes smaller for  $\vec{n} = +\vec{e}_L$  and  $\vec{n} = +\vec{e}_T$ . The  $\vec{n} = +\vec{e}_T$  polarization case gives a very small dBR/ds as compared to the unpolarized decay thus can be identified as wrong sign decay.

In Fig.(5), we plot the unpolarized  $A_{CP}$  and longitudinally and transversely polarized  $-\delta A_{CP}$  of the decay  $B \rightarrow \pi\tau^+\tau^-$ . We observe that although  $\delta A_{CP}^L$  is small,  $\delta A_{CP}^T$  is very close to  $A_{CP}$  especially in the resonance regions. Therefore, we can conclude that  $A_{CP}^L(\vec{n} = +\vec{e}_i) \simeq A_{CP}^L(\vec{n} = -\vec{e}_i)$ . The asymmetries reach to a maximum value of 13%.

## B. Numerical results of the exclusive $B \rightarrow \rho\ell^+\ell^-$ decay

In our numerical calculation for  $B \rightarrow \rho\ell^+\ell^-$  decay, we use three parameter fit of the light-cone QCD sum rule [44] which can be written in the following form:

$$F(q^2) = \frac{F(0)}{1 - a_F q^2/m_B^2 + b_F(q^2/m_B^2)^2}, \quad (60)$$

where the values of the parameters  $F(0)$ ,  $a_F$  and  $b_F$  are given in Table (II). The form factors  $A_0$  and  $A_3$  can be found from the following parametrization,

$$\begin{aligned} A_0 &= A_3 - \frac{T_3 q^2}{m_\rho m_b}, \\ A_3 &= \frac{m_B + m_\rho}{2m_\rho} A_1 - \frac{m_B - m_\rho}{2m_\rho} A_2. \end{aligned} \quad (61)$$

In Fig.(6) we present dBR/ds for the decay  $B \rightarrow \rho e^+e^-$  with unpolarized and longitudinally polarized electrons. It can be seen from this figure that the polarized spectrum for  $\vec{n} = -\vec{e}_L$  almost coincides with unpolarized spectrum, whereas the polarized  $\vec{n} = +\vec{e}_L$  spectrum is far below the unpolarized one. So, decay is naturally left handed in the SM.

In Figs.(7) and (8) we plot the longitudinally polarized CP violating asymmetries,  $A_{CP}(\vec{n})$  with  $\vec{n} = -\vec{e}_L$  and  $\vec{n} = +\vec{e}_L$ , and unpolarized  $A_{CP}$  together with the polarized quantity  $\delta A_{CP}^L$  for the decay  $B \rightarrow \rho e^+e^-$ , respectively. As can be seen from Fig.(7),  $A_{CP}(\vec{n} = -\vec{e}_L)$  is much larger than  $A_{CP}(\vec{n} = +\vec{e}_L)$ . We see from Fig.(8) that polarized CP violating



|                            | $F(0)$          | $a_F$ | $b_F$  |
|----------------------------|-----------------|-------|--------|
| $A_1^{B \rightarrow \rho}$ | $0.26 \pm 0.04$ | 0.29  | -0.415 |
| $A_2^{B \rightarrow \rho}$ | $0.22 \pm 0.03$ | 0.93  | -0.092 |
| $V^{B \rightarrow \rho}$   | $0.34 \pm 0.05$ | 1.37  | 0.315  |
| $T_1^{B \rightarrow \rho}$ | $0.15 \pm 0.02$ | 1.41  | 0.361  |
| $T_2^{B \rightarrow \rho}$ | $0.15 \pm 0.02$ | 0.28  | -0.500 |
| $T_3^{B \rightarrow \rho}$ | $0.10 \pm 0.02$ | 1.06  | -0.076 |

TABLE II:  $B \rightarrow \rho$  transition form factors in a three-parameter fit.

asymmetry  $\delta A_{CP}^L$  becomes larger than its unpolarized counterpart in some kinematic regions. Particularly, in the region  $(2m_\ell/m_B)^2 \leq s \leq ((m_{J/\psi} - 0.02)/m_B)^2$ , which is free of resonance contribution, we find that  $\delta A_{CP}^L$  is about 6%, while the unpolarized  $A_{CP}$  is about 4%. We see also from Fig.(8) that in the resonance region  $\delta A_{CP}^L$  can reach values up to 25%.

In Fig.(9), we present the dBR/ds for the decay  $B \rightarrow \rho\tau^+\tau^-$  for unpolarized, longitudinally, transversely and normally polarized  $\tau$  leptons. We see that dBR/ds for  $\vec{n} = +\vec{e}_N$  and  $\vec{n} = -\vec{e}_N$  almost coincide, while for  $\vec{n} = \pm\vec{e}_L$ , the state with  $\vec{n} = -\vec{e}_L$  is much more comparable with the unpolarized dBR/ds with respect to the one with  $\vec{n} = +\vec{e}_L$ .

In Fig.(10), we give longitudinally, transversely and normally polarized and unpolarized CP violating rate asymmetries for the decay  $B \rightarrow \rho\tau^+\tau^-$ . We observe that  $\delta A_{CP}^T$  and  $\delta A_{CP}^N$  are both smaller than  $\delta A_{CP}^L$ . Therefore, we can conclude that  $A_{CP}(\vec{n} = +\vec{e}_i) \simeq A_{CP}(\vec{n} = -\vec{e}_i)$  for  $i = T, N$ , while for  $i = L$   $A_{CP}(\vec{n} = +\vec{e}_L)$  is quite small as compared to its counterpart with  $\vec{n} = -\vec{e}_L$ .

## V. CONCLUSION

We have calculated the polarized decay rate and CP violating asymmetries of the decays  $B \rightarrow \pi\ell^+\ell^-$  and  $B \rightarrow \rho\ell^+\ell^-$ . For  $\ell = e$  which is in specific polarized channel  $\vec{n} = -\vec{e}_L$  the decay rate is comparable to the one of the unpolarized decay. The normal and the transverse polarizations are proportional to the mass of the lepton and therefore can be significant for  $\tau$  lepton only. For the  $B \rightarrow \pi\tau^+\tau^-$  decay,  $\vec{n} = \pm\vec{e}_L$  and for the  $B \rightarrow \rho\tau^+\tau^-$  decay  $\vec{n} = \pm\vec{e}_T$  and  $\vec{n} = \pm\vec{e}_N$  give similar widths. For the rest, which are defined as the wrong sign decays, the

decay rates and the CP violating asymmetries are much lower as compared to the unpolarized ones.

In conclusion, we studied the decay rate and the CP violating asymmetry of the exclusive  $B \rightarrow \pi \ell^+ \ell^-$  and  $B \rightarrow \rho \ell^+ \ell^-$  decays in the case where one of the final leptons is polarized. Since the SM is naturally left-handed, the wrong sign decays, in particular  $\vec{n} = +\vec{e}_L$  polarized  $B \rightarrow (\pi, \rho) e^+ e^-$ ,  $\vec{n} = +\vec{e}_T$  polarized  $B \rightarrow \pi \tau^+ \tau^-$  and  $\vec{n} = +\vec{e}_L$  polarized  $B \rightarrow \rho \tau^+ \tau^-$  decays, are more sensitive to new physics. Taking into account the typical branching ratios and CP violating asymmetries,  $10^{10} - 10^{11}$   $B\bar{B}$  pairs are needed for the observation of CP violation in the exclusive channels [9], which is a challenging task for the future hadron colliders. An unexpected large asymmetry in these channels and the wrong sign decays would be very significant in search for new physics beyond the SM.

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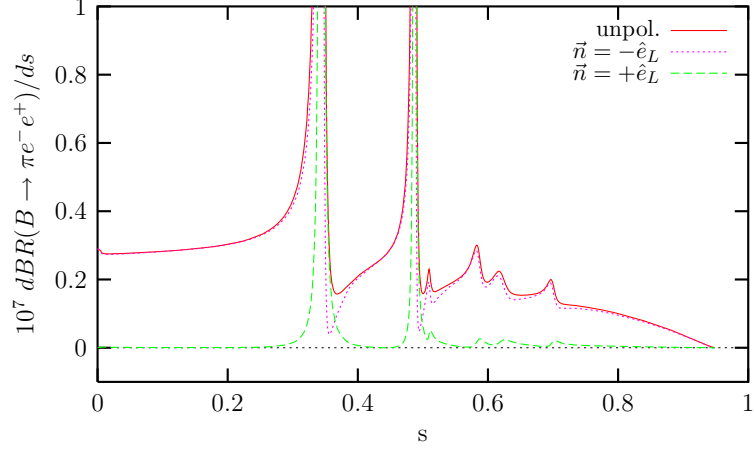


FIG. 1: Polarized and unpolarized differential branching ratios for  $B \rightarrow \pi e^+ e^-$  decay.

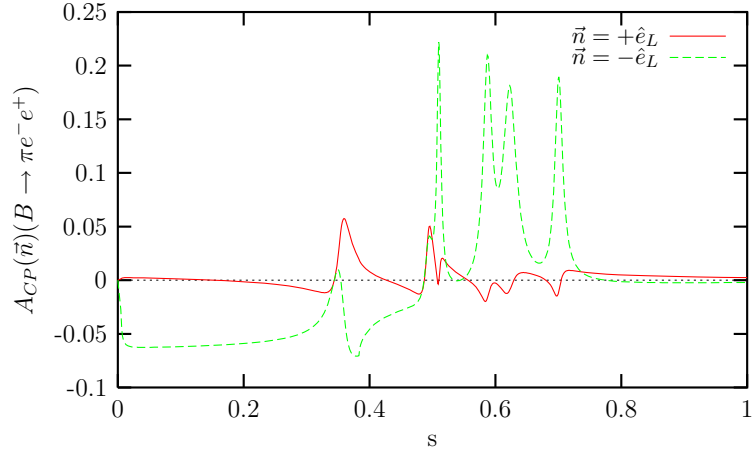


FIG. 2: Longitudinally polarized CP violating asymmetries for  $B \rightarrow \pi e^+ e^-$  decay.

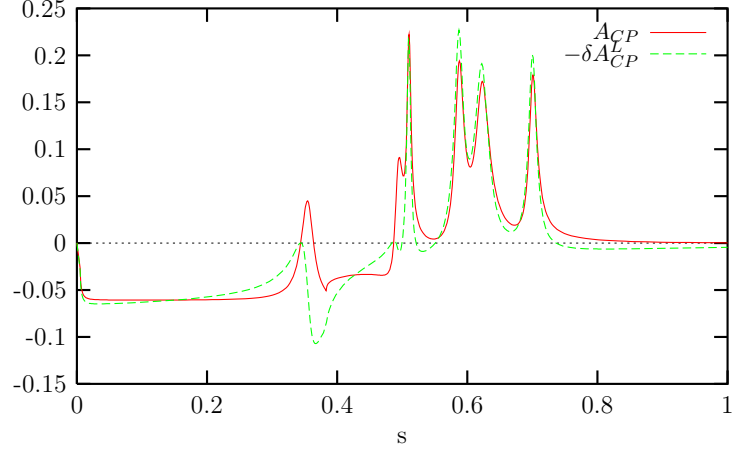


FIG. 3: Unpolarized CP violating asymmetry and longitudinally polarized quantity  $-\delta A_{CP}^L$  for  $B \rightarrow \pi e^+ e^-$  decay.

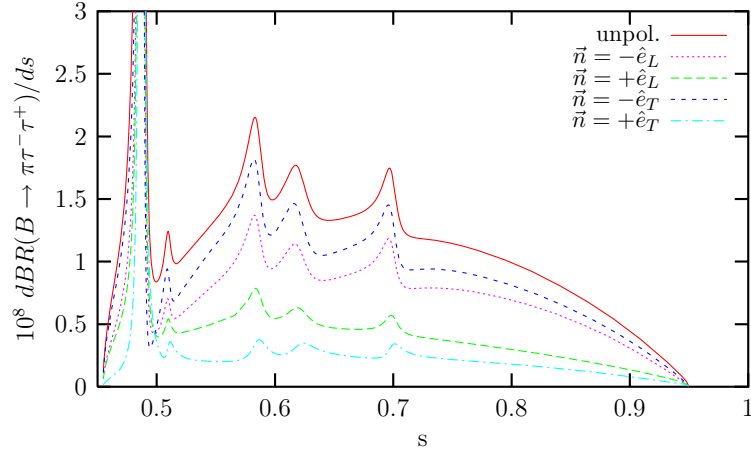


FIG. 4: Polarized and unpolarized differential branching ratios for  $B \rightarrow \pi \tau^+ \tau^-$  decay.

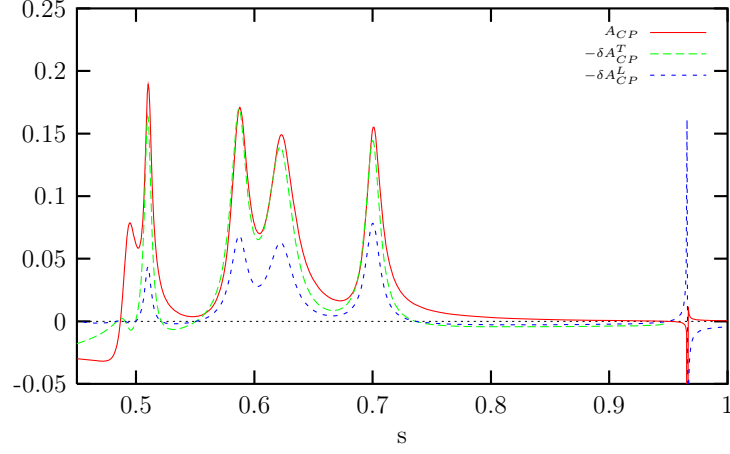


FIG. 5: Unpolarized  $A_{CP}$  and  $-\delta A_{CP}^i$  with  $i = L, T$  for  $B \rightarrow \pi \tau^+ \tau^-$  decay.

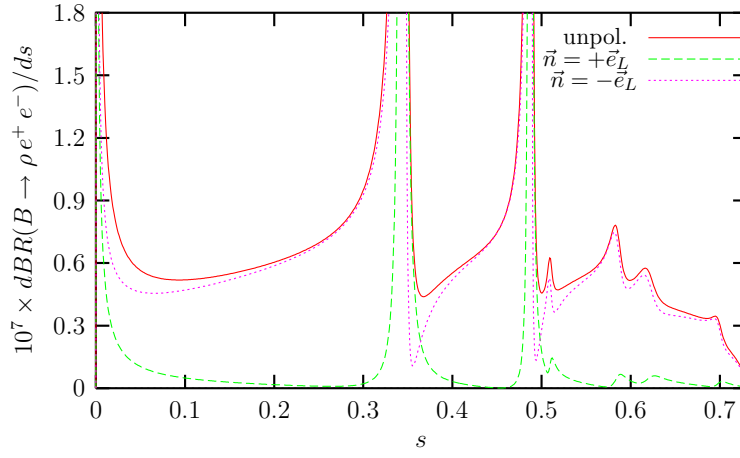


FIG. 6: Polarized and unpolarized differential branching ratios for  $B \rightarrow \rho e^+ e^-$  decay.

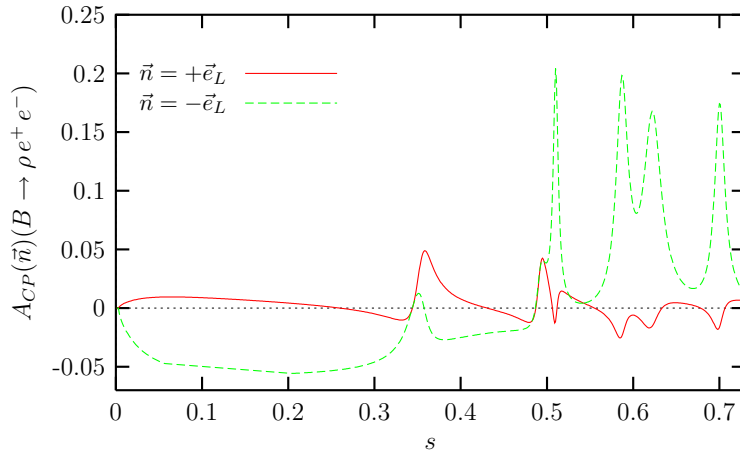


FIG. 7: Longitudinally polarized CP violating asymmetries for  $B \rightarrow \rho e^+ e^-$  decay.

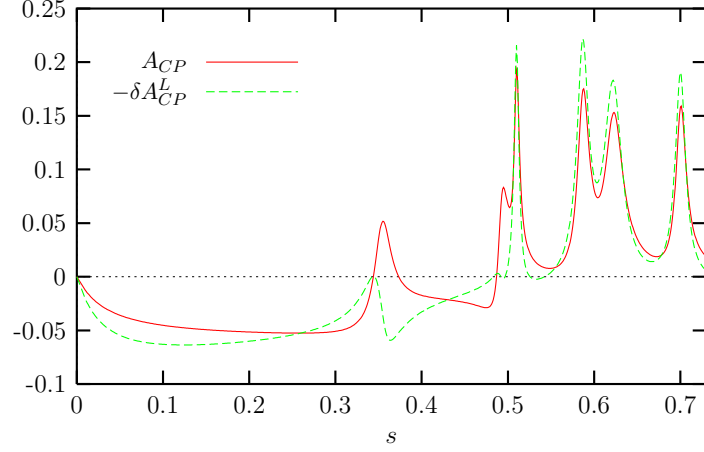


FIG. 8: Unpolarized CP violating asymmetry and longitudinally polarized quantity  $-\delta A_{CP}^L$  for  $B \rightarrow \rho e^+ e^-$  decay.

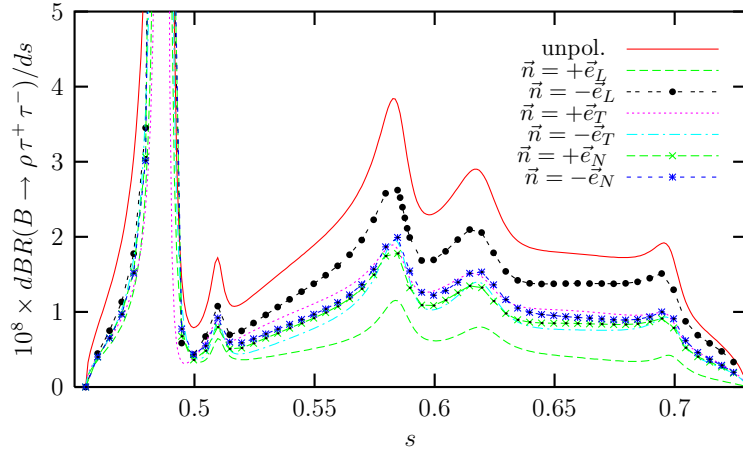


FIG. 9: Polarized and unpolarized differential branching ratios for  $B \rightarrow \rho \tau^+ \tau^-$  decay.

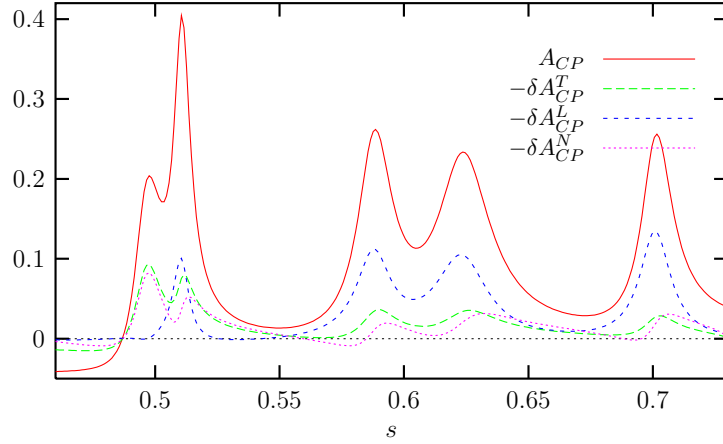


FIG. 10: Unpolarized  $A_{CP}$  and  $-\delta A_{CP}^i$  with  $i = L, T, N$  for  $B \rightarrow \rho \tau^+ \tau^-$  decay.